**Department of Mathematics**

M.Sc. (I Year/ I Sem) Question Bank

**Paper – II** (102), Subject: **Real Analysis**

**Unit – III**

1. Suppose E is a subset of a metric space (X, d) then for every n ≥ 1 ‘fn’ is a

real valued function defined on E then the sequence f1, f2, … fn … is called

a sequence of functions and it is denoted by [ ]

A)

B)

C)

D)

E) All of these

2. To each there exists a natural number no (depending on both and

the point x) such that for all and all then

sequence { is [ ]

A) Converges to ‘f’ point wise

B) Converges to f

C) Diverges to f

D) Converges to ‘f’ uniformly

E) None of these

3. Suppose fn : [0, 1]→R is defined by fn(x) = and f(x)=0

On (0,1) then this is a example of [ ]

A) fn→f as n→∞ point wise

B) fn→f as n→∞ uniformly

C) fn→f as n→∞ point wise and uniformly

D) fnf as n→∞

E) None of these

4. Suppose fn→f uniformly on E and Mn=Sup such that

for n=1, 2, 3 …….. then [ ]

A) Mn0 as n→∞

B) Mn→0 as n→∞ point wise

C) Mn→0 as n→∞

D) Mn→0 as n→∞ not uniformly

E) None of these

5. If fn : [0, 1]→R is defined by fn(x)=xn then [ ]

A) x+x2

B) x+x2+x3

C) x+x2+……..+ xn

D) x+x2+……..+ xn………

E) None of these

6. The infinite series is said to be converges uniformly if their sequence {

of nth partial sum is [ ]

A) Converges B) Diverges

C) Absolutely convergent

D) Point wise convergent

E) Uniformly convergent

7. Suppose is an infinite series of functions defined on a set E and suppose if converges then

is [ ]

A) Converges Uniformly

B) Converges Point wise

C) Diverges

D) Either Converges or Diverges

E) None of these

8. A sequence {fn} is said to be converges to f point wise on E if to each x

the sequence of real numbers is [ ]

A) Converges Uniformly to f(x)

B) Converges to f(x)

C) Diverges to f(x)

D) Absolutely converges to f(x)

E) None of these

9. The uniform limit for a sequence {fn} of continuous functions is also [ ]

A) Discontinuous

B) Removable discontinuous

C) Continuous

D) Either Continuous or discontinuous

E) None of these

10. Every polynomial p(x) is \_\_\_\_\_ [ ]

A) Continuous function

B) Discontinuous function

C) Either Continuous or discontinuous function

D) May or may not be continuous

E) None of these

11. A sequence converges uniformly on E if to each there

exists a natural number n0 such that and for

all xE where natural number is [ ]

A) Depending on point x

B) Depending only on ∈

C) Depending on both ∈ and the point x

D) Either depending on ∈ or point x

E) None of these

12. If sequence converges uniformly to f on E then the sequence

is \_\_\_\_\_ on E [ ]

A) Converges to f on E

B) Converges point wise to f on E

C) Diverges to f on E

D) Absolutely converges to f on E

E) None of these

13. Suppose is a sequence of real valued function is continuous at a

point c∈E, which is a limit point of E. if {fn} converges to f uniformly on

E then f: E→R is also [ ]

A) Uniformly Continuous

B) Discontinuous

C) Continuous

D) Converges

E) Either Continuous or Discontinuous

14. If the sequence of function fn is continuous at c∈E then [ ]

A)

B)

C)

D)

E)

15. Let α be monotonically increasing on [a, b] suppose is a sequence

of real valued functions defined on [a, b] such that on [a, b] for

n=1,2,3 ….. & suppose as n→ uniformly on [a, b] then [ ]

A) f on [a, b]

B)

C)

D)

E) f and

16. Suppose such that on [a, b] for n=1,2,3…… suppose the

series converges uniformly on [a, b] to the function f(x) then

on [a, b] and [ ]

A)

B)

C)

D) either or

E) None of these

17. Suppose f is a real valued continuous function defined on [a, b] then there is a sequence of polynomials such that [ ]

A) {Pn} converges f on [a, b]

B) {Pn} converges to point wise f on [a, b]

C) {Pn} converges to f uniformly on [a, b]

D) Converges not uniformly to f on [a, b]

E) None of these

18. A sequence {am, n} m, n>0 defined by am, n = for m = 1,2,3 …., n=1, 2, 3….. then we have [ ]

A)

B)

C)

D) =

E) =

19. Convergent series of continuous functions may have [ ]

A) Continuous sum

B) Uniformly continuous sum

C) Discontinuous sum

D) either continuous or discontinuous

E) None of these

20. Suppose is defined by then as n→ where on [0,1] is [ ]

A) Converges Point wise

B) Converges uniformly

C) Either converges point wise or uniformly

D) May or may not converges point wise

E) None of these

21. Suppose is a sequence of real valued functions defined on E and f : E→R is another function the sequence converges to f uniformly on E if and only if to each >0 there exits a natural number n0 (depending on ) such that

we have [ ]

A)

B)

C)

D)

E) None of these

22. Every convergent sequence of functions of real numbers is \_\_\_\_ on real

numbers [ ]

A) Divergent sequence

B) Cauchy sequence

C) Constant Sequence

D) Null Sequence

E) None of these

23. The compliment of [ in R is [ ]

A) (

B) (

C) (

D) (

E) (

24. Give an example of a sequence of R-integrable functions {fn} on [0, 1] such that limit of the integral is not equal to the integral of the limit [ ]

A) fn(x)=nx(1-x2)n & f(x)=0 where fn & f maps [0,1] to R

B) fn(x)=n2x(1-x2)n on [0,1]

C) Either fn(x)=nx(1-x2)n & f(x)=0 where fn & f maps [0,1] to R or

fn(x)=n2x(1-x2)n on [0,1]

D) fn(x)=nx(1-x2)n & f(x)=0 where fn & f maps [0,1] to R and

fn(x)=n2x(1-x2)n on [0,1]

E) None of these

25. A sequence of functions defined on [a, b] which is a real valued

sequence of functions is said to be Riemann integrable on [a, b] if [ ]

A) Upper integral of sequence of functions exists on [a, b]

B) Lower integral of sequence of functions exists on [a, b]

C) Upper integral and lower integral of sequence of functions exists but not equal

D) Its derivative exists

E) Upper & Lower integral of sequence of functions exists and are equal

26. The functions sinx, cosx, ex defined on real numbers are [ ]

A) Continuous functions

B) Differentiable functions

C) Bounded functions

D) Uniformly continuous

E) Continuous , Differentiable and Bounded functions

27. Give an example of convergent series of continuous functions having a

discontinuous sum [ ]

A)

B)

C)

D)

E) None of these

28. An infinite series of functions converges uniformly on its domain E if and only if to each there exists a natural number such that and each k ≥ 1 and [ ]

A)

B)

C)

D)

E) None of these

29. If sequence {Sn} of nth partial sum of {fn(x)} converges to point wise on E then the infinite series is said to be [ ]

A) Converges to point wise on E to f(x)

B) Converges to f(x) on E

C) Converges to uniformly on E to f(x)

D) Either Converges to point wise on E to f(x)or Converges to f(x) on E

E) None of these

30. If the infinite series converges to f uniformly on E then it can be written

as [ ]

A) (uniformly)

B) (uniformly)

C) (point wise)

D) Either (uniformly) or (uniformly)

E) Both (uniformly) and (uniformly)

31. Suppose sequence is a sequence of real valued functions

defined on a subset of a metric space (X, d), c is a limit point of E

& for each n ≥ 1 suppose exists if ,

converges to f:E →R uniformly on E then the sequence [ ]

A) converges

B)

C) converges uniformly

D) Both and converges uniformly

E) Both converges and

32. If the sequence real valued functions is continuous at a

point then by the definition of continuity it can be written as [ ]

A)

B)

C)

D)

E) None of these

33. Suppose { is a sequence of functions defined and differentiable

on [a, b] Let x0 such that converges , if the sequence

of derivatives converges uniformly on the interval [a, b]

then { also converges on uniformly on [a, b] to a function f(x) which is [ ]

A) Differentiable

B)

C) Both Differentiable and

D) Either Differentiable or E) None of these

34. If the function g(x) is differentiable on [x, t] then by Lagrange’s mean value theorem there exists an ‘s’ in between x and t such that [ ]

A)

B)

C)

D)

E) None of these

35. Every continuous function on the compact interval [0,1] is \_\_\_ on [0,1] [ ]

A) Discontinuous

B) Uniformly continuous

C) Removable discontinuous

D) Jump discontinuity

E) None of these

36. Every continuous sequence of functions {fn} defined on real number

system is [ ]

A) Its derivative exists

B) Not Riemann – integrable

C) Riemann-integrable

D) Either its derivative exists or not Riemann – integrable

E) None of these

37. If the function f(x)=(1-x2)n is an even function of x then [ ]

A)

B)

C)

D)

E)

38. If fn:[0,1]→R is defined by fn(x)=xn x[0, 1] fn→f as n→ point

wise on [0,1] where f:[0,1]→R is given by [ ]

A) f(x) = 0 if x∈[0,1]

B) f(x) = 1 if x=1

C) f(x) = {

D) f(x) = {

E) None of these

39. If the sequence converges to f(x) point wise then there

exists a natural number [ ]

A) Depends on the point x

B) Depends on ∈

C) Depends on both ∈ and the point x

D) Independent on x

E) Independent on ∈

40. Suppose sequence is a sequence of real valued functions defined

on the set E then their formal sum f1(x)+f2(x)+……..fn(x)….. is called \_\_\_

and it is denoted by [ ]

A) Infinite series &

B) Finite series and

C) Infinite series and

D) Finite series and

E) None of these

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**UNIT – III ANSWER KEY**

|  |  |  |
| --- | --- | --- |
| 1 | E | All of these |
| 2 | A | Converges to ‘f’ point wise |
| 3 | B | fn→f as n→∞ uniformly |
| 4 | C | Mn→0 as n→∞ |
| 5 | D | x+x2+……..+ xn……… |
| 6 | E | Uniformly convergent |
| 7 | A | Converges Uniformly |
| 8 | B | Converges to f(x) |
| 9 | C | Continuous |
| 10 | A | Continuous Function |
| 11 | B | Depending only on ∈ |
| 12 | B | Converges point wise to f on E |
| 13 | C | Continuous |
| 14 | D |  |
| 15 | E | and |
| 16 | A |  |
| 17 | C | {Pn} converges to f uniformly on [a, b] |
| 18 | B |  |
| 19 | C | Discontinuous sum |
| 20 | A | Converges Point wise |
| 21 | A |  |
| 22 | B | Cauchy sequence |
| 23 | C | ( |
| 24 | D | fn(x)=nx(1-x2)n & f(x)=0 where fn & f maps [0,1] to R and  fn(x)=n2x(1-x2)n on [0,1] |
| 25 | E | Upper & Lower integrals of sequence of functions exists and are equal |
| 26 | E | Continuous , Differentiable and Bounded function |
| 27 | A |  |
| 28 | C |  |
| 29 | A | Converges to point wise on E to f(x) |
| 30 | B | uniformly |
| 31 | E | converges ; |
| 32 | A |  |
| 33 | C | Differentiable and |
| 34 | A |  |
| 35 | B | Uniformly continuous |
| 36 | C | Riemann-integrable |
| 37 | C |  |
| 38 | D | f(x) = { |
| 39 | C | Depends on both ∈ and the point x |
| 40 | A | Infinite series & |

**UNIT – IV**

1. A Subset X of Rn is said to be a linear space if [ ]

A) x+y∈X x,y∈X

B) αx∈X ,x∈X & for all scalars

C) x+y∈Xx,y∈X and αx∈X , x∈X & for all scalars

D) Either x+y∈X x,y∈X (or) αx∈X x∈X & for all scalars

E) None of these

2. Every span is a [ ]

A) Inner product space

B) Vector Space

C) Normed Space

D) Metric Space

E) None of these

3. Let Rn be a linear space and E = {x1, x2, ……… xk} be a subset of Rn- then

E is \_\_\_ if c1x1 + c2x2 + …….. + cn xn = 0 implies c1=c2 = ……… = cn=0 [ ]

A) Linearly independent

B) Linearly dependent

C) Either linearly independent or dependent

D) both linear dependent and independent

E) None of these

4. A subset E of linear space is said to be a basis of X if [ ]

A) E is linearly independent

B) E is linearly dependent

C) Span E = X

D) E is linearly independent and span E = X

E) E linearly dependent and span E = X

5. Let r be a positive integer if a vector space X is spanned by a set of r vectors then

A) dim X = r

B) dim X < r

C) dim X ≥ r

D) dim X ≠ r

E) Dim X ≤ r

6. Let Rn = {(x1, x2, ……. xn)/ xi∈R, 1≤i≤n} then Rn is a normed space

under the norm [ ]

A) =

B) =

C) =

D) =

E) None of these

7. A mapping A of vector space X into a vector space Y is said to be a linear transformation if [ ]

A) A(x1+x2)=A(x1)+A(x2)

B) A(cx) = cAx x, x1, x2 ∈ X and all scalars

C) both A(x1+x2)=A(x1)+A(x2) and A(cx) = cAx x, x1, x2 ∈ X and all scalars

D) Either A(x1+x2)=A(x1)+A(x2) or A(cx) = cAx x, x1, x2 ∈ X and all scalars

E) None of these

8. A linear operator A on a linear space X is said to be Invertible if [ ]

A) A is one-one and onto

B) A is Many – one and onto

C) A is One – One

D) Onto

E) A is one-one and into

9. Let Rn={(x1,x2,……. xn / xi∈R, 1 ≤ i ≤ n} then Rn is an inner product space with respect to the inner product [ ]

A) d(x,y) = =

B) d(x,y) = =

C) d(x,y) = =

D) d(x,y) = =

E) None of these

10. Let Rn={(x1,x2,……. xn / xi∈R, 1 ≤ i ≤ n} then Rn is an inner product space with respect to the inner product [ ]

A) <x, y> = x.y=

B) <x, y> = x.y=

C) <x, y> = x.y=

D) <x, y> = x.y=

E) None of these

11. Every contraction mapping defined on a metric space is [ ]

A) Uniformly continuous

B) Discontinuous

C) Continuous

D) Jump discontinuity

E) Removable discontinuity

12. Let X be a metric space with metric d. A mapping is called a contraction mapping if there is a number c (0<c<1 such that for all x, y ∈ X [ ]

A)

B)

C)

D)

E)

13. Let Ω be the set of all invertible linear operators in Rn, Ω is an open subset of L(|Rn) and the mapping A→A-1 is [ ]

A) Continuous on Ω

B) Discontinuous on Ω

C) Uniformly continuous on Ω

D) Either Continuous on Ω or Uniformly continuous on Ω

E) None of these

14. Every uniformly continuous function on metric space X is [ ]

A) Discontinuous

B) Uniformly Continuous

C) Continuous

D) Either continuous or discontinuous

E) None of these

15. If X is a complete metric space and if is a contraction X into X, then there exists one and only one x∈X such that [ ]

A)

B)

C)

D)

E)

16. Let Ω be the set of all invertible linear operators on Rn if A∈Ω, B∈L(Rn) and <1 then [ ]

A) B∈Ω

B) B∉Ω

C) Either B∈Ω or B∉Ω

D) A∈Ω & B∈Ω

E) None of these

17. If A∈L(X,Yy) B∈L(Y,Z) and C=BA∈L(X,Z) where X, Y, Z are vector spaces with basis {x1, x2, ….. xn}, {y1, y2, ….. yn}, {z1, z2, ……. zn} respectively then [ ]

A) [C] = [BA] ≠ [B] [A]

B) [C] = [BA] [B] [A]

C) [C] = [BA] = [B] [A]

D) [C] = [BA][B] [A]

E) None of these

18. If A∈L(X, Y) is given by the matrix A = then [ ]

A) =

B)

C) =

D)

E) None of these

19. By Schwartz inequality if (a1, a2, ….. an), (b1, b2, …… bn) are ordered n –

tuples then [ ]

A) = .

B) .

C) .

D) = .

E) None of these

20. Suppose E is an open set in Rn and f : E→Rm is a mapping and x∈E if A1, A2 are linear transformations from Rn on to Rm satisfying

then [ ]

A) A1 ≠ A2

B) A1 ≤ A2

C) A1 ≥ A2

D) A1 = A2

E) None of these

21. If A∈L(Rn, Rm) and if x∈Rn then [ ]

A) A1(x) ≠ A

B) A1(x) = A

C) A1(x) ≤ A

D) A1(x) ≥ A

E) None of these

22. Suppose f maps on open set E⊂Rn into Rm and f is differentiable at

a point x∈E then [ ]

A) Partial derivative (Djfi) (x) exists

B) f1(x) ej = ; (1 ≤ j ≤ n);

C) f1(x) ej =

D) both (Djfi) (x) exists and f1(x) ej = ; (1 ≤ j ≤ n)

E) Either (Djfi) (x) exists or f1(x) ej = ; (1 ≤ j ≤ n)

23. Suppose f maps a convex open set E⊂Rn into Rm is differentiable in E and there is a real number M such that for every x∈E then\_\_\_ [ ]

A) = M

B) = M

C) M

D) M

E) M

24. If for a, b ∈ E any point on the line joining a, b lies in E, then we say that E is a convex set if [ ]

A)

B) (1-

C)

D)

E) None of these

25. If f is a mapping defined on a convex open set E of Rn into Rm and if f is differentiable on E such that for all x∈E then f is [ ]

A) Continuous function

B) Constant function

C) Discontinuous function

D) Uniformly continuous function

E) None of these

26. If f∈ then f is differentiable on E and the mapping x→f1(x) is continuous then by the definition of continuity for a given >0 there exists a such that

|x-y|<⇒ [ ]

A) ||f(x)-f(y)||<

B) ||(x)-(y)||<

C) ||(x)-(y)||

D) ||(x)-(y)||

E) ||f (x)-f (y)||<

27. Suppose f is a mapping defined on an open subset E of Rn into Rm then

there exits a such that |x-y|< [ ]

A) Partial derivative Djfi’s exists

B) Continuous

C) Partial derivative does not Djfi’s exists

D) Uniformly continuous

E) Partial derivative exits (Djfi’s) exists and continuous

28. Let Rn={(x1, x2, ….. xn) xi∈R , 1≤i≤n} then Rn forms a linear (vector) space under the linear operations for x=(x1, x2, ……. xn), y=(y1, y2, …… yn)∈Rn then [ ]

A) x+y=(x1+y1, x2+y2……xn+yn)

B) αx=(αx1,αx2……..αxn) for all scalars

C) both x+y=(x1+y1, x2+y2……xn+yn) and αx=(αx1,αx2……..αxn) for all scalars

D) Either x+y=(x1+y1, x2+y2……xn+yn) or αx=(αx1,αx2……..αxn) for all scalars

E) None of these

29. The set {(1,0,0), (0,1,0), (0,0,1)} of vectors in R3 is a [ ]

A) Linearly dependent Set

B) Standard basis for R3

C) Both Linearly dependent Set and Standard basis for R3

D) Linearly independent set

E) None of these

30. If in a vector space Rn, A is a set of vectors containing zero vector then

A is [ ]

A) Linearly independent

B) A basis of Rn

C) Subspace of Rn

D) Linearly dependent

E) None of these

31. If in a vector space Rn, A is a set of vector does not containing zero vector

then A is [ ]

A) Linearly independent

B) Linearly dependent

C) Both Linearly independent and Linearly dependent

D) Either Linearly independent or Linearly dependent

E) None of these

32. Let Rn be an n-dimensional vector space i.e., dim Rn=n then [ ]

A) Every set of n-vectors forms a basis of Rn

B) Every set of n-vectors which spans Rn is a basis of Rn

C) Every set of n-vectors which is linearly independent is a basis of Rn

D) Both Every set of n-vectors forms a basis of Rn and Every set of n-vectors which spans Rn is a basis of Rn is true

E) None of these

33. If A, B are invertible transformation then (AB)-1 = [ ]

A) A-1B-1 B) BA-1

C) B-1A

D) B-1A-1

E) None of these

34. A linear space X is said to have dimension r if [ ]

A) X contains an independent set of r elements

B) X does not contains an independent set of r+1 elements

C) both X contains an independent set of r elements and X does not contains an independent set of r+1 elements are true

D) Either X contains an independent set of r elements or X does not contains an independent set of r+1 elements is true

E) None of these

35. Let Rn be a linear space if x1, x2, …. xk∈Rn and c1, c2, ….. ck are scalars then the vector c1x1 + c2 x2 + ……. + ck xk is called \_\_\_\_\_\_ of x1, x2, ….. xk [ ]

A) Linear span

B) Linear combination

C) Basis of Rn

D) Linearly independent

E) Linearly dependent

36. Let Rn be a vector space and a subset E={x1, x2, ….. xk} then E is said to be linearly dependent if their exits scalars [ ]

A) not all zero’s

B) such that c1 x1 + c2 x2 + ……. + ck xk = 0

C) Scalars are all zero’s

D) not all zero’s such that c1 x1 + c2 x2 + ……. + ck xk = 0

E) None of these

37. A mapping f defined on an open set E of Rn onto Rm is said to be continuously differentiable if [ ]

A) f is differentiable at each point of E

B) the mapping x→f1(x) is a continuous mapping of E into L(Rn, Rm)

C) Both f is differentiable at each point of E and the mapping x→f1(x) is a continuous mapping of E into L(Rn, Rm)

D) f is differentiable at any point of E

E) Both the mapping x→f1(x) is a continuous mapping of E into L(Rn, Rm) and

f is differentiable at any point of E are true

38. If B={x1, x2, ……… xk} is a basis of X then every element of X is uniquely expressed as a \_\_\_\_\_\_\_ of elements of B [ ]

A) Linear combination

B) Linear span

C) Linearly dependent set

D) Linearly independent set

E) Co-ordinate

39. If x = c1x1+c2x2+….. + cn xn is the unique representation of x∈Rn interms of elements x1, x2, ………. xn of a basis B then c1, c2, …… ck are called as \_\_\_\_\_\_ w.r.t. the basis B [ ]

A) Scalars of x

B) Constant

C) Co-ordinate of x

D) Basis of x

E) None of these

40. Under the norm define =Sup{x∈Rn, then the space L(Rn, Rm) satisfies [ ]

A)

B) for all A, B ∈L(Rn, Rm)

C) for all A∈L(Rn, Rm) and for all scalars c

D) All of these

E) None of these

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**UNIT – IV ANSWER KEY**

|  |  |  |
| --- | --- | --- |
| 1 | C | x+y∈X x,y∈X and αx∈X , x∈X & for all scalars |
| 2 | B | Vector Space |
| 3 | A | Linearly independent |
| 4 | D | E is linearly independent and span E = X |
| 5 | E | Dim X ≤ r |
| 6 | A | = |
| 7 | C | both A(x1+x2)=A(x1)+A(x2) and A(cx) = cAx x, x1, x2 ∈ X and all scalars |
| 8 | A | A is one-one and onto |
| 9 | B | d(x,y) = = |
| 10 | D | <x, y> = x.y= |
| 11 | C | Continuous |
| 12 | A |  |
| 13 | A | Continuous on Ω |
| 14 | C | Continuous |
| 15 | D |  |
| 16 | A | B∈Ω |
| 17 | C | [C] = [BA] = [B] [A] |
| 18 | D |  |
| 19 | B | . |
| 20 | D | A1 = A2 |
| 21 | B | A1(x) = A |
| 22 | D | both (Djfi) (x) exists and f1(x) ej = ; (1 ≤ j ≤ n) |
| 23 | E | M |
| 24 | D |  |
| 25 | B | Constant function |
| 26 | B | ||(x)-(y)||< |
| 27 | E | Partial derivative exits (Djfi’s) exists and continuous |
| 28 | C | both x+y=(x1+y1, x2+y2……xn+yn) and αx=(αx1,αx2……..αxn) for all scalars |
| 29 | B | Standard basis for R3 |
| 30 | D | Linearly Dependent |
| 31 | A | Linearly independent |
| 32 | C | Every set of n-vectors which is linearly independent is a basis of Rn |
| 33 | D | B-1A-1 |
| 34 | C | both X contains an independent set of r elements and X does not contains an independent set of r+1 elements are true |
| 35 | B | Linear combination |
| 36 | D | not all zero’s such that c1 x1 + c2 x2 + ……. + ck xk = 0 |
| 37 | C | Both f is differentiable at each point of E and the mapping x→f1(x) is a continuous mapping of E into L(Rn, Rm) |
| 38 | A | Linear combination |
| 39 | C | Co-ordinate of x |
| 40 | D | All of these |