

**Faculty of Science**  
**B.Sc (Mathematics) III-Year, CBCS -VI Semester**  
**Regular Examinations -June/July, 2022**  
**PAPER: Analytical Solid Geometry**

Time: 3 Hours

Max Marks: 80

**Section-A**

- I. Answer any eight of the following questions (8x4=32 Marks)
- Find the value of  $K$  for which the plane  $x + y + z = K\sqrt{3}$  touches the sphere  $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$ .
  - Find the Centre and radius of the circle  $x^2 + y^2 + z^2 = 25$ ,  $2x + y + 2z = 9$
  - If the radius of the sphere  $x^2 + y^2 + z^2 + 6x - 8y - t = 0$  is 6 then find  $t$ .
  - Find the enveloping cone of the sphere  $x^2 + y^2 + z^2 + 2x - 4y = 0$  with vertex at  $(1,1,1)$ .
  - Find the equation of the cone with vertex  $(1,1,0)$  and guiding curve  $x^2 + y^2 = 4$ ,  $z = 0$ .
  - Find the equation to the cone which passes through the three coordinate axes and the lines  $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$  and  $\frac{x}{3} = \frac{y}{-1} = \frac{z}{1}$ .
  - Find the equation to the right circular cylinder whose guiding circle is  $x^2 + y^2 + z^2 = 25$ ,  $x - y + z = 3$ .
  - Find the equation of the cylinder whose generators are parallel to  $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$  and passing through the curve  $3x^2 + 2y^2 = 1, z = 0$ .
  - Define right circular cone and cylinder.
  - Find the equations of tangent planes to  $7x^2 - 3y^2 - z^2 + 21 = 0$  which passes through the line  $7x - 6y + 9 = 0, z = 3$ .
  - The plane  $lx + my + nz = p$  touch the conicoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$  then show that  $a^2l^2 + b^2m^2 - c^2n^2 = p^2$
  - Show that  $3x^2 + 4y^2 + z^2 - x + 12y - 4z + 13 = 0$  is an ellipse and find its Centre.

**Section-B**

- II. Answer the following (4x12=48 Marks)
- (a) Find the equation of the sphere through the points  $(0,-2,-4), (2,-1,-1)$  and whose Centre lies on the line  $2x - 3y = 0 = 5y + 2z$ .  
(OR)  
(b) Show that the spheres  $x^2 + y^2 + z^2 = 25, x^2 + y^2 + z^2 - 24x - 40y - 18z = -225$  touch externally. Find the point of contact.
  - (a) Show that the equation  $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$  represents a cone with vertex  $(-7/6, 1/3, 5/6)$ .  
(OR)  
(b) Show that the plane  $ax + by + cz = 0$  cuts the cone  $yz + zx + xy = 0$  in

Perpendicular lines if  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

15. (a) Find the equation of the enveloping cylinder of the sphere  $x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$  having its generators parallel to the line  $x = y = z$ .

(OR)

- (b) Find the equation to the right circular cylinder whose axis is  $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$  and radius 2.

16. (a) Find the locus of the points from which three mutually perpendicular tangent lines can be drawn to the conicoid  $ax^2 + by^2 + cz^2 = 1$

(OR)

- (b) A point P moves so that the section of enveloping cone of the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  with P as vertex by XY plane is a circle. Show that P lies on

One of the conics  $\frac{y^2}{b^2 - a^2} + \frac{z^2}{c^2} = 1, x = 0$  (or)  $\frac{x^2}{a^2 - b^2} + \frac{z^2}{c^2} = 1, y = 0$

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## Faculty of Science

## B.Sc(Mathematics) III-Year, CBCS –VI Semester Backlog Examinations –Jan, 2023

## PAPER: Analytical Solid Geometry

Time: 3 Hours

Max Marks: 80

## Section-A

- I. Answer any *eight* of the following questions (8x4=32 Marks)
- Find the centre and radius of the sphere  $x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$ .
  - Find the pole of the plane  $x - y + 5z - 3 = 0$  with respect to the sphere  $x^2 + y^2 + z^2 = 9$ .
  - Find the equation of the sphere through the circle  $x^2 + y^2 + z^2 = 0$ ,  $2x + 3y + 4z = 5$  and the point  $(1,2,3)$ .
  - Find the equation of the cone whose vertex is  $(1,1,0)$  and guiding curve is  $x^2 + z^2 = 4$ ,  $y = 0$ .
  - Find the equation of the cone which passes through the three coordinate axis and the lines  $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ ,  $\frac{x}{3} = \frac{y}{-1} = \frac{z}{1}$ .
  - Find the intersecting points of the cone  $4x^2 - y^2 + z^2 = 0$  and the line  $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z-1}{2}$ .
  - Find the equation to the cylinder whose generators are parallel to  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  and guiding curve is  $x^2 + y^2 = 1$ ,  $z = 1$ .
  - Find the right circular cylinder of radius 1 and axis is the line  $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$ .
  - Find the equation of the cylinder with generators parallel to Z-axis and passing through the curve  $ax^2 + by^2 = 2z$ ,  $lx + my + nz = p$ .
  - Find the intersecting points of the line  $\frac{x+5}{-3} = \frac{y-4}{1} = \frac{z-11}{7}$  with the conicoid  $12x^2 - 17y^2 + 7z^2 = 7$ .
  - Find the equations to the tangent planes to  $7x^2 - 3y^2 - z^2 + 21 = 0$  which passes through the line  $7x - 6y + 9 = 0$ ,  $z = 3$ .
  - Show that the plane  $3x + 12y - 6z - 17 = 0$  touches the conicoid  $3x^2 - 6y^2 + 9z^2 + 17 = 0$  and find their point of contact.

## Section-B

- II. Answer the following questions (4x12=48 Marks)
- (a) Find the equation of the sphere through the four points  $(4, -1, 2)$ ,  $(0, -2, 3)$ ,  $(1, -5, -1)$  and  $(2, 0, 1)$ .  
(OR)  
(b) Find the limiting points of the co-axial system defined by the spheres  $x^2 + y^2 + z^2 + 3x - 3y + 6 = 0$ ,  $x^2 + y^2 + z^2 - 6z + 6 = 0$ .
  - (a) Show that the general equation of the cone which touches the three co-ordinate planes is  $\sqrt{fx} + \sqrt{gy} + \sqrt{hz} = 0$ .  
(OR)  
(b) Prove that the equation  $x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6zx + 8x - 19y - 2z - 20 = 0$  represents a cone and find its vertex.
  - (a) Find the equation of a right circular cone whose vertex is  $(1,1,1)$ , axis is  $\frac{x-1}{-1} = \frac{y-1}{2} = \frac{z-1}{3}$  and semi vertical angle is  $30^\circ$ .  
(OR)  
(b) Find the equation to the right circular cylinder of radius 2, whose axis is the line  $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{6}$ .
  - (a) Find the equations to the tangent planes to the surface  $4x^2 - 5y^2 + 7z^2 + 13 = 0$  parallel to the plane  $4x + 20y - 21z = 0$  find their point of contact also.  
(OR)

- (b) Find the locus of the perpendiculars from the origin to the tangent planes to the surface  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  which cut off from its axes intercepts. The sum of whose reciprocals is equal to the constant  $1/k$ .

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Time: 3 Hours

Max Marks: 80

## Section-A

- I. Answer any *eight* of the following questions (8x4=32 Marks)
- Find the centre and radius of the sphere  $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$ .
  - Find the tangent plane to the sphere  $x^2 + y^2 + z^2 - 6x + 2z + 1 = 0$  at the point  $(2, -2, 1)$ .
  - Find the equation of the sphere whose end points of the diameter are  $(2, 3, -1)$  and  $(1, -2, -1)$ .
  - Find the equation of the cone whose vertex is the origin and guiding curve is  $x^2 + y^2 = 4, z = 2$ .
  - Find the intersecting points of the cone  $11x^2 - 5y^2 + z^2 = 0$  and the line  $\frac{x+1}{-1} = \frac{y-12}{5} = \frac{z-7}{2}$ .
  - Find the equation of the cone which passes through the three coordinate axes and the lines  $\frac{x}{1} = \frac{y}{-3} = \frac{z}{3}, \frac{x}{1} = \frac{y}{-1} = \frac{z}{-1}$ .
  - Find the equation of the cylinder whose generators are parallel to  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  and the guiding curve is  $x^2 + y^2 = 16, z = 0$ .
  - Find the right circular cylinder of radius 2 and axis is the line  $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$ .
  - Find the equation of the cylinder with generators parallel to Z-axis and passing through the curve  $ax^2 + by^2 = 2z, lx + my + nz = p$ .
  - Find the points of intersection of the line  $\frac{x+5}{-3} = \frac{y-4}{1} = \frac{z-11}{7}$  with the conicoid  $12x^2 - 17y^2 + 7z^2 = 7$ .
  - Find the equations to the tangent planes to  $7x^2 - 3y^2 - z^2 + 21 = 0$  which passes through the line  $7x - 6y + 9 = 0, z = 3$ .
  - Find the tangent planes to the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  which are parallel to the plane  $lx + my + nz = 0$ .

## Section-B

- II. Answer the following questions (4x12=48 Marks)
- (a) Show that the points  $(5,0,2), (2, -6,0), (7, -3,8), (4, -8,6)$  are concyclic.  
(OR)  
(b) Show that the spheres  $x^2 + y^2 + z^2 = 25, x^2 + y^2 + z^2 - 24x - 40y - 18z + 225 = 0$  touch externally and find the point of contact.
  - (a) Prove that the equation  $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$  represents a cone whose vertex is  $(-1, -2, -3)$ .  
(OR)  
(b) Prove that the cones  $x^2 - y^2 + 2z^2 - 3yz + 4zx - 5xy = 0$  and  $17x^2 + 8y^2 + 29z^2 + 28yz - 46zx - 16xy = 0$  are reciprocal.
  - (a) Find the equation of the right circular cone which passes through the point  $(1,1,2)$  and has vertex at the origin and axis is the line  $\frac{x}{2} = \frac{-y}{4} = \frac{z}{3}$ .  
(OR)  
(b) Find the equation of the right circular cylinder of radius 2, whose axis is the line  $\frac{x-1}{2} = y - 2 = \frac{z-3}{2}$ .
  - (a) Find the equations to the tangent planes to the surface  $4x^2 - 5y^2 + 7z^2 + 13 = 0$  parallel to the plane  $4x + 20y - 21z = 0$  find their point of contact also.

(OR)

- (b) If the section of the enveloping cone of the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  whose vertex is P by the plane  $z = 0$  is a rectangular hyperbola. Show that the locus of P is

$$\frac{x^2 + y^2}{a^2 + b^2} + \frac{z^2}{c^2} = 1.$$

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