R-19

Code:4308/R

Faculty of Science

B.Sc (Mathematics) II-Year, CBCS –IV Semester

Regular Examinations -June/July, 2022

PAPER: Algebra

Time: 3 Hours | Max Marks: 80

Section – A

I. Answer any *eight* of the following questions. (8×4=32 Marks)

1. Prove that $(ab)^2 = a^2b^2$ in a group G if $ab = ba$ for all $a, b \in G$
2. Prove that center of a group G is a subgroup of G.
3. Find the inverse of the element $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in $GL(2, z_{11})$ with respect to matrix multiplication.
4. Prove that a group of prime order is cyclic.
5. Determine whether the following is even or odd $\alpha = (12)(134)(152)$
6. Write the permutation (12)(13)(23)(142) as a product of disjoint cycles.
7. Find all units of Z_{14}.
8. Prove that the intersection of subring of a ring R is a subring of R.
9. Every quotient group of an abelian group is abelian.
10. Show that the correspondence $x \to 5x$ from Z_5 to Z_{10} does not preserve addition.
11. Define prime ideal and maximal ideal with examples.
12. If $Z_6 = \{0,1,2,3,4,5\}$, A={0,3} then find all the elements of factor ring Z_6/A

Section – B

II. Answer **all** the questions. (4×12=48 Marks)

13. (a) Prove that every subgroup of cyclic group is cyclic.

(OR)

(b) Define group. Let $G=\{\begin{bmatrix} a & a \\ a & a \end{bmatrix}$, a ϵ R, a≠0}. Show that G is a group under matrix multiplication.

14. (a) State and Prove Lagrange theorem

(OR)

(b) State and prove Orbit-Stabilizer theorem.

15. (a) What is the order of the element $14 + \langle 8 \rangle$ in the factor group $\frac{Z_{14}}{\langle 8 \rangle}$

(OR)

(b) Find all the solutions of $x^2 - x + 2 = 0$ over $Z_3[i]$.

16. (a) Define maximal ideal of a ring if R is a commutative ring with unity and M is a maximal ideal of R, Then prove that R/M is field.

(OR)

(b) Prove that if ϕ be a ring homomorphism from a ring R to a ring S then $\phi(A)$ is an ideal of S

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R-19

Code:4308/BL/19

Faculty of Science

B.Sc(Mathematics)II-Year, CBCS –IV Semester Backlog Examinations –Jan, 2023

PAPER: Algebra

Time: 3 Hours | Max Marks: 80

Section – A

I. Answer any *eight* of the following questions. (8×4=32 *Marks*)

1. For any two elements a, b in a group G, Prove that $(ab)^{-1} = b^{-1}a^{-1}$.
2. Find the inverse of the element $\begin{bmatrix} 4 & -4 \\ -4 & 3 \end{bmatrix}$ in $SL(2, Z_5)$ with respect to matrix multiplication.
3. Prove that $H \cap K$ is a subgroup of G if H and K are subgroup of G.
4. Determine whether the following is even or odd permutation $a = (1\ 2)(1\ 3\ 4)(1\ 5\ 2)$.
5. Find the order of the permutation (531)(2468)(135)
6. Let H be any subgroup of G, and let $a, b \in G$ then prove that $aH = H$ *iff* $a \in H$.
7. Prove that intersection of two normal subgroup is a normal subgroup.
8. Let R be a ring. The center of R is the set $S = \{x \in R: ax = xa, \forall a \in R\}$. Then show that S is a subring of R.
9. Find all units of Z_{14}.
10. Show that $\phi : C \to C$ given by $\phi(a + ib) = a - ib$ is a ring homomorphism.
11. Define prime ideal and maximal ideal with examples.
12. Let $f(x) = 4x^3 + 2x^2 + x + 3$, and $g(x) = 3x^4 + 3x^3 + 3x^2 + x + 4$ are twopolynomials in $Z_5[x]$. Compute $f(x) + g(x)$ and $f(x).g(x)$.

Section – B

II. Answer the following questions. (4×12=48 Marks)

13. (a) G is a group, a is an element in G with order n, k is a positive integer then show that $< a^k > = < a^{gdc(n,k)} >$, $|a^k| = \frac{n}{gcd(n,k)}$

    (OR)

    (b) If a be an element of a group G and let $|a| = 15$ then find all generators of G and also compute the orders of a^3, a^6, a^9, a^{10} of G.
14. (a) State and Prove Lagrange's theorem.

    (OR)

    (b) State and Prove Orbit-Stabilizer theorem.
15. (a) Let ϕ be a homomorphism from a group G to a group $\overline{G}$. Then prove that $Ker\ \phi$ is a normal subgroup of G.

    (OR)

    (b)Construct multiplication table for $Z_3[i]$.
16. (a) Let R be a commutative ring with unity and A be an ideal of R. Then prove that R/A is an integral domain if and only if A is a prime ideal of$R$.

    (OR)

    (b) Prove that if ϕ be a ring homomorphism from a ring R to a ring S then $\phi(A)$ is an ideal of S.

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R-19

Code:4308/19/REG

Faculty of Science

B.Sc (Mathematics) II-Year, CBCS –IV Semester Regular Examinations –June, 2023

PAPER: ALGEBRA

Time: 3 Hours Max Marks: 80

Section – A

I. Answer any *eight* of the following questions (8×4=32 Marks)

1. Solve the followings

   (a) $7 +_{10} x = 4 \text{ in } Z_{10}$ (b) $x +_{12} 7 = 11 \text{ in } Z_{12}$

2. Prove that center of the group $Z(G)$ is a subgroup of G
3. What are all the generators of Z_{25}?
4. Compute all the cosets of $8Z$ in Z
5. Let $H = \{0, \pm 3, \pm 6, \pm 9, \ldots\}$. Find all the left cosets of H in Z?
6. Determine whether the following is even or odd $\alpha = (1234)(3521)$
7. Prove that the centre of any group is a normal subgroup.
8. Find all units of $Z \times Z$, where Z is the ring of integers.zz
9. Prove that intersection of two normal subgroup is a normal subgroup.
10. If $Z_6 = \{0,1,2,3,4,5\}, A = \{0,3\}$ then find all the elements of factor ring Z_6/A
11. Define prime ideal and maximal ideal with examples.
12. Show that $\phi : C \to C$ given by $\phi(a + ib) = a - ib$ is a ring homomorphism.

Section – B

II. Answer **all** the questions. (4×12=48 *Marks*)

13. (a) Prove if H a non-empty finite subset of a group G. If H is closed under the operation of G, then H is a subgroup of G.

    (OR)

    (b) If a be an element of a group G and *let* $|a| = 15$ then find all generatorsof G and also compute the orders of a^3, a^6, a^9, a^{10} of G.

14. (a) (i) State and Prove Lagrange's theorem.

    (ii) Let G be a group of order 100. Find the number of subgroups of order3 in G.

    (OR)

    (b) State and Prove Orbit-Stabilizer theorem.

15. (a) Construct multiplication table for $Z_3[i]$.

    (OR)

    (b) Determine all homomorphisms from Z_{12} *to* Z_{30}

16. (a) Prove that if ϕ be a ring homomorphism from a ring R to a ring S then $\phi(A)$ is an ideal of S.

    (OR)

    (b) Let R be a commutative ring with unity and A be an ideal of R. Then prove that R/A is an integral domain if and only if A is a prime ideal of R.

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