R-19

Code: 5308/19/BL

Faculty of Science

B.Sc. (Mathematics) III-Year, CBCS-V Semester

Backlog Examinations -June/July, 2022

PAPER: Linear Algebra

Time: 3 hours | Max Marks: 80

Section-A

I. Answer any *eight* of the following (8x4=32 Marks)

1. Prove that intersection of two subs pace is a sub space.

2. If $w = \left\{ \begin{bmatrix} 5b + 2c \\ b \\ c \end{bmatrix} / b, C \in R \right\}$ prove that W is a subspace of R^3

3. Prove that $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis of R^3

4. Find the characteristic polynomial of $A = \begin{bmatrix} 6 & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{bmatrix}$

5. If a 3x8 matrix A has rank 3 find dim Nul A, dim Row A and Rank A^T

6. Is $\lambda = -2$ an Eigen value of $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$

7. Prove that the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable over the field C.

8. Find the complex Eigen values of $A = \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}$

9. If $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$ find D^{10}

10. Let $\overline{y} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ and $\overline{u} = \begin{pmatrix} 7 \\ 1 \end{pmatrix} \epsilon R^2$ find orthogonal projection of $\overline{y}$ onto the line through $\overline{u}$ and the origin.

11. Show that the set $\left\{ \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right), \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right) \right\}$ forms an orthonormal set in R^3

12. Compute the unit vector in the direction of $\begin{bmatrix} \frac{7}{4} \\ \frac{1}{2} \\ 1 \end{bmatrix}$

Section-B

II. Answer all of the following (4x12=48 Marks)

13. (a) Let $\bar{v}_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix}$, $\bar{v}_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$, $\bar{v}_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$ Determine if $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ is a basis for R^3

(OR)

(b) State and prove spanning set theorem

14. (a) If $\bar{v}_1, \bar{v}_2, \ldots \ldots \bar{v}_n$ are Eigen vectors corresponding to distinct Eigen values $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$ of (nxn) matrix A then the set $\{\bar{v}_1, \bar{v}_2, \ldots\ldots.\bar{v}_n\}$ is linearly independent.

(OR)

(b) Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

15. (a) Diagonalize the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

(OR)

(b) Let $T: p_3 \to p_4$ be the transformation that maps a polynomial $p(t)$ into the polynomial $p(t) + t^2\, p(t)$.

(i) Find image of $p(t) = 2 - t + t^2$

(ii) Show that T is a linear transformation

(iii) Find the matrix T relative to the bases

$B = \{1, t, t^2\}$ and $C = \{1, t, t^2, t^3, t^4\}$

16. (a) Define orthonormal set and prove that the set $\left\{ \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix} \right\}$ is orthonormal set.

(OR)

(b) In the vector space $R^3(R)$ construct on orthonormal basis using Gram-Schmidt orthogonalization process from the basis $\{(3,4,0), (2,1,-1), (-2,1,3)\}$

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R-19

Code:5308/R

Faculty of Science

B.Sc (Mathematics) III-Year, CBCS –V Semester

Regular Examinations –Jan, 2023

PAPER: Linear Algebra

Time: 3 Hours | Max Marks: 80

Section-A

I. Answer any *eight* of the following questions (8x4=32 Marks)

1. Define vector space and sub space of a vector space.
2. Show that W= $\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a - 3b - c = 0 \right\}$ is a sub space of $\mathbb{R}^3$.
3. Find a basis for the set of vectors in $\mathbb{R}^3$ in the plane $x - 3y + 2z = 0$.
4. State the Rank theorem. If A is a 7x9 matrix with a two dimensional null space, what is the rank of A?
5. $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ is an eigen vector of $\begin{bmatrix} -4 & 3 & 3 \\ 2 & -3 & -2 \\ -1 & 0 & -2 \end{bmatrix}$ then find the eigen value.
6. Define eigen value and eigen vector of a square matrix A.
7. Let A= PDP^{-1} where $P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ then compute A^4.
8. Show that the eigen values of A= $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ are $a \pm bi$ with corresponding eigen vectors $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ and $\begin{bmatrix} 1 \\ i \end{bmatrix}$ where $a, b \in \mathbb{R}$.
9. Find the $\mathcal{B}$-matrix for the transformation $X \mapsto AX$ where $\mathcal{B}=\{b_1, b_2\}$, $A = \begin{bmatrix} -4 & -1 \\ 6 & 1 \end{bmatrix}, b_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$
10. Define orthogonal set and orthonormal basis.
11. $w = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$ and $x = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$ are two vectors then find $w.w$ and $x.x$
12. If u and v are orthogonal vectors then prove that $\| u + v \|^2 = \| u \|^2 + \| v \|^2$

Section-B

II. Answer the following questions (4x12=48 Marks)

13.(a) (i)Prove that null space of an mxn matrix A is a sub space of $\mathbb{R}^n$.

(ii) Find basis and dimension of the sub space $\left\{ \begin{bmatrix} p - 2q \\ 2p + 5r \\ -2q + 2r \\ -3p + 6r \end{bmatrix} : p, q, r \in \mathbb{R} \right\}$

(OR)

(b) Define coordinate vector relative to the basis $\mathcal{B}$. $\mathcal{B} = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $p(t) = 1 + 4t + 7t^2$ relative to $\mathcal{B}$.

14.(a) Find basis for the row space, the column space and the null space of the matrix A = $\begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$

(OR)

(b)(i) Define characteristic equation of a square matrix. Find eigen values of

$A=\begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$

(ii) Define similar matrices. Prove that if A and B are similar matrices then they have same characteristic polynomial.

15.(a) Diagonalize the matrix $A=\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ if possible.

(OR)

(b) $T: \mathbb{P}_2 \rightarrow \mathbb{R}^3$ is defined by $T(p) = \begin{bmatrix} p(-1) \\ p(0) \\ p(1) \end{bmatrix}$ then show that T is a linear transformation and find the matrix for T relative to the basis $\{1, t, t^2\}$ for $\mathbb{P}_2$ and the standard basis for $\mathbb{R}^3$.

16.(a)(i) State and prove parallelogram law.

(ii) Construct an orthonormal basis for $\begin{bmatrix} 3 \\ -4 \\ 6 \end{bmatrix}, \begin{bmatrix} -3 \\ 14 \\ 7 \end{bmatrix}$

(OR)

(b) Determine the set of vectors $\left\{ \begin{bmatrix} 1/\sqrt{18} \\ 4/\sqrt{18} \\ 1/\sqrt{18} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix} \right\}$ are orthonormal?

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Faculty of Science

B.Sc. (Mathematics) III-Year, CBCS-V Semester Backlog Examinations –June, 2023

PAPER: LINEAR ALGEBRA

Time: 3 hours   Max Marks: 80

Section-A

I. Answer any EIGHT of the following questions (8x4=32 Marks)

1. Define vector space and subspace of a vector space.
2. If $\{v_1, v_2, v_3, \dots \dots v_p,\}$ are p vectors in a vector space V, then show that span $\{v_1, v_2, v_3, \dots \dots v_p,\}$ is a subspace of V.
3. Let $A = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$ and $W = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Determine if W is in Col A? Is W is in Nul A?
4. If a 4x7 matrix A has rank 3, find dim Nul A, dim Row A and rank A^T.
5. Find basis and dimension for the subspace $\left\{ \begin{bmatrix} s-2t \\ s+t \\ 3t \end{bmatrix} : s, t \in \mathbb{R} \right\}$.
6. Find the dimension of the subspace of all vectors in $\mathbb{R}^3$ whose 1st and 2nd entries are equal.
7. Define eigen value and eigen vector. Find eigen values of $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.
8. Show that $\lambda = 1$ is an eigen value for the matrix $A = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$ also find corresponding eigen vector.
9. Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ are u and v eigen vectors of A?
10. Compute the orthogonal projection of $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ on to the line through $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ and the origin.
11. Let $u = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, v = \begin{bmatrix} -7 \\ -4 \\ 6 \end{bmatrix}$ then compute $u.v, ||u||^2$ and $||u+v||^2$.
12. State and prove Parallelogram law in an inner product space.

Section-B

II. Answer all of the following questions (4x12=48 Marks)

13.(a) Define coordinate vector relative to the basis B. Find the coordinate vector $[\overline{X}]_B$ relative to the basis $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} \right\}$ where $X = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$

(OR)

(b) Define basis of a vector space. Let $v_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix}, v_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}, v_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$

Determine if $\{v_1, v_2, v_3,\}$ is a basis for $\mathbb{R}^3$

14.(a) Find dimensions of the Nul A and Col A where $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

(OR)

(b) Find basis for the subspace $\{(a,b,c): a - 3b + c = 0, b - 2c = 0, 2b - c = 0\}$ also find dimension.

15.(a) Assume the mapping $T: p_2 - p_2$ defined by

$T(a_0 + a_1 t + a_2 t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$ is linear.

Find the matrix representation of T relative to the basis $B = \{1, t, t^2\}$.

(OR)

(b) Diagonalize the matrix $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

16.(a) Define orthogonal set and orthonormal set. Show that the set

$\left\{ \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{20} \\ 3/\sqrt{20} \end{bmatrix}, \begin{bmatrix} 3/\sqrt{10} \\ -1/\sqrt{20} \\ -1/\sqrt{20} \end{bmatrix}, \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \right\}$ is orthogonal or not? Explain.

(OR)

(b) Construct an orthonormal basis for $W = \left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ 9 \\ -9 \\ 3 \end{bmatrix} \right\}$ by using Gram-Schmidt process.

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