**R-19**

Code:3308/BL

## Faculty of Science
## B.Sc (Mathematics) II Year, CBCS - III Semester Backlog
## Examinations -June/July, 2022
## PAPER: Real Analysis

Time: 3 Hours (Max Marks: 80)

### Section – A

I. Answer any *eight* of the following questions. (8×4=32 Marks)

1. Write out the first five terms of the following sequences.
   (a) $s_n = \frac{1}{3n+1}$ (b) $b_n = \frac{3n+1}{4n+1}$ (c) $c_n = \frac{n}{3^n}$ (d) $d_n = sin\left(\frac{n\pi}{4}\right)$
2. Prove $\lim \frac{1}{n^2} = 0$
3. Prove that convergent sequences are Cauchy sequences.
4. Let $f(x) = 2x^2 + 1$ for $x \in R$. Prove that $f$ is continuous on $R$ by using $\epsilon, \delta$ definition.
5. Prove that if $f$ is continuous at $x_0$ and $g$ is continuous at $f(x_0)$, then the composite function $g\,o\,f$ is continuous at $x_0$.
6. Prove that if $f$ and $g$ be real-valued functions that are continuous at $x_0$ in *R*. Then (i) $f + g$ is continuous at $x_0$; (ii) $fg$ is continuous at $x_0$;
7. Prove that if $f$ is differentiable at a point a, then $f$ is continuous at a.
8. State and prove Rolle's theorem.
9. Apply L'Hospital's rule and find the limits for
   (a) $\lim_{x \to 0} \frac{cosx-1}{x^2}$ (b) $\lim_{x \to \infty} \frac{x^2}{e^{3x}}$
10. Prove that if $f$ is a bounded function on $[a,b]$, and if $P$ and $Q$ are partitions of $[a,b]$, then $L(f,P) \le U(f,Q)$.
11. Prove that every continuous function $f$ on $[a,b]$ is integrable.
12. Prove that if $f$ integrable functions on $[a,b]$, and let $c$ be a real number. Then show that $cf$ is integrable and $\int_a^b cf = c\int_a^b f$

### Section – B

II. Answer the following questions. (4x12=48 Marks)

13. (a) (i) Prove that every monotone bounded sequence is convergent.
    (ii) Prove that if a sequence $(s_n)$ converges to $s$, then every subsequence of it also converges to $s$.

    (OR)

    (b) State and prove Boltzano Weiestrass Theorem.
14. (a) Let $f$ be a one-to-one continuous function on an interval $I.$ Then $f$ is strictly increasing $[x_1 < x_2$ implies $f(x_1) < f(x_2)]$ or strictly decreasing $[x_1 < x_2$ implies $f(x_1) > f(x_2)]$

    (OR)

    (b) If $f$ is continuous on a closed interval $[a,b]$, then $f$ is uniformly continuous on $[a,b]$.
15. (a) State and prove Lagrange's mean value theorem.

    (OR)

    (b) Let $f$ and $g$ be functions that are differentiable at the point 'a' then prove that each of the functions $cf$ $[c\ a\ constrant]$, $f+g, fg$ and $f/g$ is also differentiable at a, if $g(a) \neq 0$.
16. (a) Prove that a bounded function $f$ on $[a,b]$ is integrable if and only if for each $\epsilon > 0$ there exists a partition *P* of $[a,b]$ such that $U(f,P) - L(f,P) < \epsilon$

    (OR)

    (b) State and prove first fundamental theorem of Integral calculus.

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**Faculty of Science**

**B.Sc(Mathematics)II-Year, CBCS–III Semester Regular Examinations –Jan, 2023**

**PAPER: Real Analysis**

Time: 3 Hours | Max Marks: 80

## Section-A

I. Answer any *eight* of the following questions (8x4=32 Marks)

1. Define limit of a sequence and Cauchy sequence.
2. Prove Lim sup$|s_n|$=0 if and only if Lim $s_n$=0.
3. Let $(s_n)$= (0,1,2,1, 0,1,2,1, 0,1,2,1, .........) and $(t_n)$= (2,1,1,0, 2,1,1,0, 2,1,1,0, .........) be two sequences then find Lim sup $(s_n+t_n)$ and Lim inf $(s_n+t_n)$.
4. Prove $f(x) = |x|$ is continuous function on $\mathbb{R}$.
5. If $f(x)$ is continuous at $x_0$ then show that $|f(x)|$ is continuous $x_0$.
6. Let $f(x)$=4 for $x \geq 0$, $f(x) = 0$ for $x < 0$ and $g(x)$= $x^2\ \forall x$. Determine $f+g,\ f.g$.
7. Let $f(x)$= $x^2$ for $x \geq 0$, $f(x) = 0$ for $x < 0$. Show that $f$ is differentiable at $x = 0$.
8. Write the statements of Rolle's theorem and generalized mean value theorem.
9. Prov that every differentiable function at $x = a$ is continuous at $x = a$.
10. Define Riemann integral of a function.
11. Find lower and upper Darboux integrals for $f(x) = 1$ for $x \in \mathbb{Q}$ and $f(x) = 0$ for $x \in \mathbb{R} - \mathbb{Q}$ on the interval [a, b].
12. Let $f$ be integrable function on [a, b] and $c \in \mathbb{R}$ then show that $cf$ is integrable.

## Section-B

II. Answer the following questions (4x12=48 Marks)

13. (a)(i) Prove that every Cauchy sequence is bounded.

    (ii) Test the convergence of the series $\sum\frac{n-1}{n^2}$

    (OR)

    (b) State and prove Ratio test in infinite series.

14. (a)(i) Show that $f(x) = 3x + 11$ is uniformly continuous on $\mathbb{R}$.

    (ii)Prove that if $f$ is continuous on [a,b] then $f$ is uniformly continuous on [a,b]

    (OR)

    (b) Prove that every continuous function on [a, b] is bounded on [a, b].

15. (a) State and prove Lagrange's mean value theorem.

    (OR)

    (b)(i) Find $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x$

    (ii) Find $\lim_{x\to 0}\frac{x^3}{sinx-x}$

16. (a) State and prove 1$^{st}$ fundamental theorem of calculus.

    (OR)

    (b) Prove that every continuous function $f$ on [a, b] is integrable.

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**Faculty of Science**

**B.Sc (Mathematics) II-Year, CBCS –III Semester Backlog Examinations –June, 2023**

**PAPER: Real Analysis**

Time: 3 Hours Max Marks: 80

**Section-A**

I. Answer any EIGHT of the following questions (8x4=32 Marks)

1. By using definition of the limit of a sequence prove that $\lim_{n\to\infty} \frac{2n-1}{3n+2} = \frac{2}{3}$
2. Calculate $\sum_{n=1}^{\infty}(2/3)^n$
3. Define sub sequence and Infinite series.
4. Discuss the continuity of the function $g(x) = \begin{cases} -1 \text{ for } x<0 \\ 0 \text{ for } x=0 \\ 1 \text{ for } x>0 \end{cases}$ at $x_0 = 0$.
5. Define continuous and uniform continuous functions.
6. If a function $f(x)$ is continuous at $x_0$ then show that $|f(x)|$ is continuous at $x_0$.
7. Prove $|\cos x - \cos y| \le |x-y| \ \forall x,y \in \mathbb{R}$.
8. Prove that every differentiable function is continuous at a point $x_0$.
9. Find $\lim_{x\to 0}(1+2x)^{1/x}$
10. Let $f(x) = \begin{cases} x \text{ for all } x \in \mathbb{Q} \\ 0 \text{ for all } x \in \mathbb{R}-\mathbb{Q} \end{cases}$ then calculate upper and lower Darboux sums for $f(x)$ on the interval [0,b].
11. If $f$ and $g$ are integrable functions on [a,b] and $f(x) \le g(x)\ \forall x \in$ [a,b] then show that $\int_a^b f(x) \le \int_a^b g(x)$.
12. Define Riemann integral.

**Section-B**

II. Answer the following questions (4x12=48 Marks)

13.(a)(i) State and prove squeeze theorem for sequences.

(ii) Prove $\lim_{n\to\infty}(a)^{1/n}$ =1 for $a>0$.

(OR)

(b) State and prove Root-test for infinite series.

14.(a) (i) Prove that the function $f(x) = x^2$ is continuous at $x_0 = 2$ by using $(\varepsilon, \delta)$ property.

(ii) Prove $|x|$ is continuous function on $\mathbb{R}$.

(OR)

(b) (i) Prove every continuous function defined on [a,b] is uniform continuous.

(ii) Let $f(x) = \sqrt{4-x}$ for $x \le 4$ and $g(x) = x^2$ for all $x \in \mathbb{R}$ then find $fog(0), gof(0)$ and $fog(1)$.

15.(a) state and prove Rolle's theorem.

(OR)

(b) (i) Find $\lim_{x\to 0}\frac{1+\cos x}{e^x-1}$ (ii) Find $\lim_{x\to 0}\frac{e^{2x}-\cos x}{x}$

16.(a) Prove that every monotonic function $f(x)$ on [a,b] is integrable.

(OR)

(b) State and prove fundamental theorem of calculus II.

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