

Faculty of Science

B. Sc (Mathematics) I-Year, CBCS –I Semester Backlog Examinations, January 2021
PAPER: DIFFERENTIAL AND INTEGRAL CALCULUS

Time: 2 Hours

Max Marks: 80

I. Answer Any FOUR from the following questions (4x20=80 Marks)

1. If $4 = \sin^{-1} \frac{x^2+y^2}{x+y}$, S/T $x \frac{\partial u}{\partial x} + y \frac{\partial v}{\partial y} = \tan 4$

2. If $4 = \log (x^3 + y^3 + z^3 - 3xyz)$, then S/P $(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})^2 4 = \frac{-9}{(x+y+z)^2}$

3. Find maximum value of $f(x, y, z) = x^a y^b z^c$ subject to the condition
 $x + y + z = 1$, where a, b, c are positive integers.

4. Expand the function $f(x, y) = x^2 + xy - y^2$ by tan cos threesome in powers of $(x - 1)$ and $(y + 2)$

5. If a curve is defined by the equation $x = f(t)$ and

$$y = \phi(t), \text{ then P/T } p = \frac{(x^{12} + y^{12})^{3/2}}{x^{11}y^{11} - 4^{11}x^{11}}$$
 where denote differentiation w.r.t.

6. For a cycloid $x = a(t + \sin t)$, $y = (1 - \cos t)$, prove that $e = 4a \cos(\frac{1}{2}t)$

7. Find the volume of the solid obtained by revolving the log of the curve
 $a^2 y^2 = x^2(2a - x)(x - a)$ about 'x' axis.

8. S/T the length of the curve $x^2 = a^2(1 - e^{y/a})$ measured from

$$0(0,0) \text{ to } p(x,y) \text{ is } a \log \left(\frac{a+x}{a-x} \right) - x$$

Faculty of Science

B. Sc (Mathematics) I-Year, CBCS –I Semester

Backlog Examinations -June/July, 2022

PAPER: Differential and Integral Calculus

Time: 3 Hours

Max Marks: 80

Section-A

I. Answer any eight from the following (8x4=32 Marks)

1. If $u = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y}$, $xy \neq 0$, p/t $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$
2. Find second order partial derivatives of e^{x-y}
3. Verify Euler's theorem for $z = (x^2 + xy + y^2)^{-1}$
4. If $H = f(y-z, z-x, x-y)$, p/t $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$
5. Find the maximum & minimum values of the function $f(x) = 8x^5 - 15x^4 + 10x^3$
6. If $z = x^2 + y^2$, $x = at^2$ and $y = 2at$, then evaluate $\frac{dz}{dt}$
7. Find the envelope of the straight lines $x \cos \alpha + y \sin \alpha = l \sin \alpha \cos \alpha$, α being the parameter.
8. Find the radius of curvature at the origin of the curve $x^3 - 2x^2y + 3xy^2 - 4y^3 + 5x^2 - 6xy + 7y^2 - 8y = 0$
9. Find the envelope of the family of curve $y = mx + am^3$
10. Find the length of the arc of the curve $y = \log \sec x$ from $x = 0$ to $x = \pi/3$
11. Find the perimeter of the cardioid $r = a(1 - \cos \theta)$
12. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$

Section-B

II. Answer the following questions (4x12=48 Marks)

13. (a) If $z = f(x, y)$ is a homogenous function of x, y of degree n , then p/t

$$x \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y \frac{\partial^2 z}{\partial y^2} = n(n-1)z$$

(OR)

(b) If $u = \log(x^2 + y^3 + z^3 - xyz)$, then s/t $(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})^2 u = \frac{-9}{(x+y+z)^2}$

14. (a) Expand the function $f(x, y) = x^2 + xy - y^2$ by Taylor's theorem in powers of $(x-1)$ and $(y+2)$

(OR)

(b) In a plane triangle, find the maximum value of $u = \cos A \cos B \cos C$ where A, B, C are the angles of the triangle.

15. (a) For a cycloid $x = a(t + \sin t)$, $y = a(1 - \cos t)$, p/t $e = 4a \cos(\frac{1}{2}t)$

(OR)

(b) Find the evolutes of the hyperbola $2xy = a^2$

16. (a) Find the volume of the solid obtained by revolving one arch of the cycloid $x = a(0 + \sin \theta)$, $y = a(1 + \cos \theta)$

(OR)

(b) Find the volume of the solid obtained by revolving the loop of the curve $a^2 y^2 = x^2(2a - x)(x - a)$ about the x -axis.

Faculty of Science

B.Sc (Mathematics)I-Year, CBCS–I Semester Backlog Examinations –June, 2023

PAPER: Differential and Integral Calculus

Time: 3 Hours

Max Marks: 80

Section-A

I. Answer any *eight* of the following questions (8x4=32 Marks)

- If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$
- $z = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then find $\frac{\partial z}{\partial x} \frac{\partial z}{\partial y}$
- Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$ does not exist.
- If $H = f(y-z, z-x, x-y)$ prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$
- If $u = x^2 - y^2, x = 2r - 3s + 4, y = -r + 8s - 5$ find $\frac{\partial u}{\partial r}$
- Find the radius of curvature of $y = 4\sin x - \sin 2x$ at $P = \left(\frac{\pi}{2}, 4\right)$
- Find the radius of curvature of $y = xe^{-x}$ at its maximum point.
- By Newton's method find the radius of curvature of $y = x^4 - 4x^3 - 18x^2$ at $(0,0)$
- Find radius of curvature of $s = 4a \sin \frac{\psi}{3}$ at $P = \left(\frac{\pi}{2}, 2a\right)$
- Find the length of the arc of the curve $y = \log \sec x$ from $x = 0$ to $x = \frac{\pi}{3}$
- Find the length of the arc of the curve $x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$ from $\theta = 0$ to $\theta = \pi$
- Find the perimeter of the cardioid $r = a(1 - \cos \theta)$ from $\theta = 0$ to $\theta = \pi$

Section-B

II. Answer the following questions (4x12=48 Marks)

- (a) (i) State and Prove Euler's theorem for Homogeneous functions.
(ii) If $x^y = y^x$ then find $\frac{\partial y}{\partial x}$

(OR)

- (b) If $f(x,y) = \begin{cases} \frac{x^3y}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & (x,y) = (0,0) \end{cases}$ then show that $f_{yx}(0,0) \neq f_{xy}(0,0)$

- (a) If $z = e^u f(v), u = ax + by, v = ax - by$, then show that $b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz$

(OR)

- (b) Expand $e^x \log(1+y)$ in powers of x and y

- (a) Find the centre and circle of curvature at the point $P\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$

(OR)

- (b) Find the evolute of the hyperbola $2xy = a^2$

- (a) Prove that the loop of the curve $x = t^2, y = t - \frac{t^3}{3}$ is of length $4\sqrt{3}$

(OR)

- (b) Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x -axis.
