

TELANGANA UNIVERSITY
S.S.R. DEGREE COLLEGE, NIZAMABAD (C.C:5029)
III SEMESTER INTERNAL ASSESSMENT II EXAMINATIONS
MATHEMATICS (REAL ANALYSIS) QUESTION BANK

I. Multiple choice questions.

1. The derivative of function $g(x)=x^2$ is $g'(x) =$ [d]
 (a) x^2+1 (b) x (c) 0 (d) $2x$
2. If $f(x)=x^n$ then $f'(x) =$ [c]
 (a) x^n-a^n (b) $x^{1/2}$ (c) $n.x^{n-1}$ (d) $n.x^n$
3. $\lim_{x \rightarrow a} x^k =$ [b]
 (a) k (b) a^k (c) k^2 (d) a
4. If f and g are functions that are differentiable at point 'a' then $(f+g)'(a) =$ [d]
 (a) $f'(a) g'(a)$ (b) $f'(a) g(a)$ (c) $f'(a)-g'(a)$ (d) $f'(a)+ g'(a)$
5. If $f(x)=\sin(x^3)$ then $f'(x)=$ [a]
 (a) $(3x^2) \cos(x^3)$ (b) $\cos(x^3)$ (c) $3\cos x^3$ (d) $3x \cos x$
6. The function e^x on \mathbb{R} is [a]
 (a) Strictly increasing (b) Strictly decreasing (c) Bounded (d) Continuous
7. The function $\cos x$ on $[0, \pi]$ is [d]
 (a) Increasing (b) Continuous (c) Differentiable (d) Strictly decreasing
8. If $f(y) = y^{1/n}$, n is even then domain of f is [b]
 (a) $(0,0)$ (b) $[0, \infty)$ (c) $(0,00)$ (d) $(-1,0)$
9. $\lim_{x \rightarrow 0^+} \log x =$ [a]
 (a) $-\infty$ (b) 0 (c) 1 (d) None
10. The limit $\lim_{x \rightarrow \infty} \left[1 - \frac{1}{x}\right]^x$ is indeterminate of the form [c]
 (a) 0^0 (b) ∞^0 (c) 1^∞ (d) $\frac{0}{0}$
11. The necessary and sufficient condition for Riemann integrability of bounded function f on $[a, b]$ with a partition p . for every $\varepsilon > 0$ is, [a]
 (a) $0 \leq U(P, f) - L(P, f) < \varepsilon$ (b) $0 \leq U(P, f) - L(P, f) > \varepsilon$
 (c) $U(P, f) - L(P, f) = \varepsilon$ (d) None of these
12. Let f is a bounded function defined on a bounded interval $[a, b]$, then corresponding to any partition P of $[a, b]$ [a]
 (a) $L(f,P) \leq U(f,P)$ (b) $L(f, P) U(f, P)$ (c) Both (a) and (b) (d) Neither (a) nor (b)
13. Let f be a bounded function defined on $[a, b]$ and P be the partition on $[a, b]$ and Q is refinement of P . then [a]
 (a) $L(Q, f) \geq L(P, f)$ (b) $L(Q, f) \leq L(P, f)$ (c) $U(Q, f) \geq U(P, f)$ (d) None of these
14. Let f be a bounded function on $[a, b]$ and p be the partition of $[a, b]$ and Q is refinement of p then, [b]
 (a) $L(Q, f) \leq L(P, f)$ (b) $U(Q, f) \leq U(P, f)$ (c) $U(Q, f) \geq u(P, f)$ (d) None of these
15. If f is Riemann integrable on $[a, b]$, then [d]
 (a) $\left| \int_a^b f(x) dx \right| = \int_a^b |f(x)| dx$ (b) $\left| \int_a^b f(x) dx \right| \geq \int_a^b |f(x)| dx$
 (c) $\left| \int_a^b f(x) dx \right| \neq \int_a^b |f(x)| dx$ (d) $\left| \int_a^b f(x) dx \right| \leq \int_a^b |f(x)| dx$
16. Let f be a bounded function defined on $[a, b]$, then f' is Riemann integrable if and only if [c]
 (a) $\int_a^b f \leq \int_a^{\bar{b}} f$ (b) $\int_a^b f \geq \int_a^{\bar{b}} f$ (c) $\int_a^b f = \int_a^{\bar{b}} f$ (d) $\int_a^{\bar{b}} f = -\int_a^b f$

17. Let f, g be bounded functions on $[a, b]$ and let P be partition on $[a, b]$, then, [a]

(a) $U(P, f+g) \leq U(P, f)+U(P, g)$ (b) $U(P, f+g) \geq U(P, f)+U(P, g)$

(c) $U(P, f+g) = U(P, f)+U(P, g)$ (d) $L(P, f+g) \leq L(P, f)+L(P, g)$

18. Let f be a continuous function on $[a, b]$ and ϕ be a differentiable function on $[a, b]$ such that $\phi(x) = f(x) \forall x \in [a, b]$ then [b]

(a) $\int_a^b f(x)dx = \phi(a) - \phi(b)$ (b) $\int_a^b f(x)dx = \phi(b) - \phi(a)$

(c) $\int_a^b f(x)dx = \phi(a) + \phi(b)$ (d) None of these

19. If f be a bounded function on $[a, b]$ and if P, Q are partitions of $[a, b]$, $P \subseteq Q$ then [a]

(a) $L(f, P) \leq L(f, Q) \leq U(f, Q) \leq U(f, P)$ (b) $L(f, P) \geq L(f, Q) \leq U(f, Q) \geq U(f, P)$

(c) $L(f, P) \leq L(f, Q) \geq U(f, Q) \geq U(f, P)$ (d) $L(f, P) \geq L(f, Q) \geq U(f, Q) \geq U(f, P)$

20. If f and $|f|$ are integrable on $[a, b]$, then [b]

(a) $\left| \int_a^b f \right| \geq \int_a^b |f|$ (b) $\left| \int_a^b f \right| \leq \int_a^b |f|$ (c) $\left| \int_a^b f \right| = \int_a^b |f|$ (d) None of these

II. Fill in the Blanks

1. The domain of 'f' is the set of points at which f is differentiable.

2. If f is differentiable at a point a, then f is continuous at 'a'.

3. If f and g are functions that are differentiable at the point 'a' then $(cf)'(a) = \underline{c.f'(a)}$

4. If x_0 and y_0 are both end point of $[a, b]$, then f is a function.

5. If f is a differentiable function on an interval (a,b) then f is strictly increasing if $f'(x) > 0$

6. The form of $R_n(x) = \frac{(x-y)^{n-1}}{(n-1)!} f^{(n)}(y).x$ is known as cauchy's form of remainder

7. The derivative of $f(x)=x+2$ at $x=a$ is 1

8. If $f(x)=\sin x$ and $g(x) = \frac{1}{2}$ then composite function $(f \circ g)(x) = \underline{\sin \frac{1}{x}}$

9. $\lim_{n \rightarrow \infty} \left[1 - \frac{1}{n} \right]^n = \underline{e^{-1}}$

10. The limit $\lim_{x \rightarrow 0^+} x^x$ is of the indeterminate form 0^0

11. If P and Q are two partitions of a closed and bounded interval $[a, b]$, then Q is called refinement of P
If $Q \supset P$

12. If P_1 and P_2 be two partitions on $[a, b]$ and $P = P_1 \cup P_2$, then p is called common refinement of P_1 and P_2

13. Let f be a real valued bounded function defined on $[a, b]$ and f is called Riemann integrable on $[a, b]$

If $\int_a^b f = \int_a^b f$

14. The lower Riemann integral of f over $[a, b]$ is supremum of $L(P, f)$ over all partitions, $P \in [a, b]$.

15. If $f: [a, b] \rightarrow \mathbb{R}$ is a bounded function, then $L(P, -f) = \underline{-U(P, f)}$

16. The upper Riemann integral of f over $[a, b]$ is the infimum of $U(P, f)$ over all partitions $P \in [a, b]$.

17. Let f be a real valued bounded function on $[a, b]$ then, the lower Riemann integral of f cannot exceed the upper Riemann integral of f over $[a, b]$

18. A function $F(x)$, whose derivative $F'(x) = f(x)$ is called an anti-derivative of $f(x)$.

19. Let f be continuous on $[a, b]$ and $F(x) = \int_a^x f(t) dt \forall x \in [a, b]$ then $F'(x) = \underline{f(x)} \forall x \in [a, b]$

20. Let f be Riemann integrable on $[a, b]$ and ϕ be a differentiable function on $[a, b]$ such that $\phi'(x) = f(x)$

$\forall x \in [a, b]$, then $\int_a^b f(x) dx = \underline{(b)-(a)}$